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Stability of winding cosmic wall lattices with X type junctions

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Abstract. This work confirms the stability of a class of domain wall lattice models that can produce accelerated cosmological expansion, with pressure to density ratio $w = -1/3$ at early times, and with $w = -2/3$ at late times when the lattice scale becomes large compared to the wall thickness. For walls of tension T_I , the relevant X type junctions could be unstable (for a sufficiently acute intersection angle α) against separation into a pair of Y type junctions joined by a compound wall, only if the tension T_{II} of the latter were less than $2T_I$ (and for an approximately right-angled intersection if it were less than $\sqrt{2}T_I$) which can not occur in the class considered here. In an extensive category of multicomponent scalar field models of forced harmonic (linear or non-linear) type it is shown how the relevant tension – which is the same as the surface energy density U of the wall – can be calculated as the minimum (geodesic) distance between the relevant vacuum states as measured on the space of field values Φ^i using a positive definite (Riemannian) energy metric $dU^2 = \tilde{G}_{ij} d\Phi^i d\Phi^j$ that is obtained from the usual kinetic metric (which is flat for a model with ordinary linear kinetic part) by application of a conformal factor proportional to the relevant potential function V . For suitably periodic potential functions there will be corresponding periodic configurations – with parallel walls characterised by incrementation of a winding number – in which the condition for stability of large scale bunching modes is shown to be satisfied automatically. It is suggested that such a configuration – with a lattice lengthscale comparable to intergalactic separation distances – might have been produced by a late stage of cosmological inflation.

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1. Introduction

The motivation for recent interest [1, 2, 3, 4, 5] in the stability properties of various kinds of domain wall lattice has arisen from the suggestion of Bucher and Spergel [6, 7] that some such lattice might account for the observed acceleration of the expansion of the universe. Attention has been drawn to the important qualitative distinction between the X type of (crossover) junction that is more favorable [1, 2, 3] for stability of the lattice, and the Y type of junction, in which there is no freedom of adjustment in the equilibrium angle of intersection (which must be $\pi/3$ if the wall tensions are all equal).

For an X type crossover the most symmetric possibility is a right angle intersection, but for opposing pairs with equal tension equilibrium will still be possible when there is a positive deviation δ so that the walls meet at an acute angle

$$\alpha = \pi/2 - \delta, \quad (1)$$

It has however been emphasized [5] that in order to contribute to a stable lattice such an X type equilibrium must be stable against decomposition into a pair of Y type junctions (see Fig. 1), a requirement that was not explicitly checked in the particular toy field models I suggested [1] as examples in my original discussion of this subject, and that has been called into doubt [5] in the particular case of what will be referred to here as the pentavac doublet model.

In order to address the lattice stability question, the first task of the present work is to provide a simple general criterion for the stability of such an X type equilibrium junction. A strategy for testing this criterion will then be provided for an extensive class of forced harmonic field models, including the pentavac doublet model [1], for which it is confirmed that (contrary to the doubts that have been expressed [5]) the stability condition is indeed satisfied. The main part of this work is concerned with the related but more delicate issue of stability against bunching of parallel walls, which will be dealt with in the same framework.

Although – like many other possibilities such as the monovac triplet model developed at the end of this article – the pentavac doublet model can provide a regular lattice that is stable, it should be mentioned that it is nevertheless unsatisfactory from the point of view of the purpose that motivated its introduction, which was to provide a random lattice of the kind to which my (still unproven) five colour conjecture was concerned[1]. The special feature of the pentavac doublet model (see Fig. 2 at the end) is the admission of simple domain walls between any of the 10 pairs that can be chosen from its 5 distinct but equivalent vacua, and the admission of X type crossovers involving any of the 5 possible combinations of 4 distinct vacua. However, for each such combination, the pentavac model allows only one of the 3 mathematically conceivable ways of choosing the diagonally opposing pairs, whereas all of them would be needed for a random solution of the five colour problem.

This limitation on all the cases investigated so far – namely that they provide lattices that can be stable only when sufficiently regular, not random, and hence that they depend on some prerequisite “tuning” of the universe – considerably reduces the attraction of such domain wall models in comparison, for example, with the simple hypothesis of an appropriately “tuned” cosmological constant.

Despite this limitation, wall scenarios of the periodic lattice type considered here remain viable in principle, as a possibility that should perhaps be taken more seriously, particularly

[4] if the evidence for cosmological anisotropy [8] is confirmed. In the systems envisaged here, the collective stability of the walls depends on their feature of having topological winding numbers, which must add up coherently with the same sign, whereas a random system (such as would be expected from a Kibble type symmetry breaking mechanism [9, 10, 11]) would contain roughly equal numbers of positive and negative windings (which in the long run would undergo mutual destruction, leaving a residue of string anomalies).

In comparison with the (unavoidable) problem of accounting for the observed (but still mysterious [12]) preponderance of ordinary matter over antimatter, the analogous, but numerically less extreme, problem of accounting for the required preponderance (on a sufficiently large scale) of positive over negative walls appears to be less intractable. The reasoning developed below suggests that it may be soluble on the basis of statistical fluctuations in the framework of suitable inflationary models – apparently without any need for special parameter tuning beyond what was already inherent in such models.

2. Simple and compound wall configurations in models with X type junctions.

The recently raised issue[5] of possible instability of X type junctions arises in bosonic field models of the kind I suggested as candidates for providing domain wall lattices, in which the essential feature – as illustrated in Fig. 2 – is the existence in the space of classical field values of neighbouring subsets of four equivalent discrete vacua – meaning minima of the potential energy V – separated by four ridges of higher energy that meet in a cross shaped configuration at a peak of even higher energy. Using the letters A, B, C, D, to label the energy minima in cyclic order, as one goes round the peak of V in configuration space, entails the concomitant notation AB, BC, CD, DA to label the ridges that separate them.

For each such ridge in the space of field values, there will be a corresponding flat domain wall equilibrium state in ordinary spacetime: for example AB will designate the – energy minimising – domain wall state specifiable as a function of a single cartesian coordinate, x , say by the condition that it tends to the field configuration A as $x \rightarrow -\infty$ and to field configuration B as $x \rightarrow +\infty$, with maximum energy density at $x = 0$. Since there is nothing to break the Lorentz invariance in directions parallel to the $x = 0$ plane, such a wall will be a Dirac type brane, with isotropic tension, T_I equal to its surface energy density.

As well as such simple branes associated with the field space energy ridges AB, BC, CD, DA, there may also be compound wall configurations separating non adjacent vacuum pairs, for which the possible combinations are AC and BD. In order to be stable against splitting into the relevant pair of simple walls (for example in order for the compound wall AC to be stable against splitting into the separate simple walls AB and BC) the field model should evidently be such that the energy density (and tension) T_{II} say of the compound wall configuration – if it exists – is not greater than twice that of a single wall:

$$T_{II} \leq 2T_I \tag{2}$$

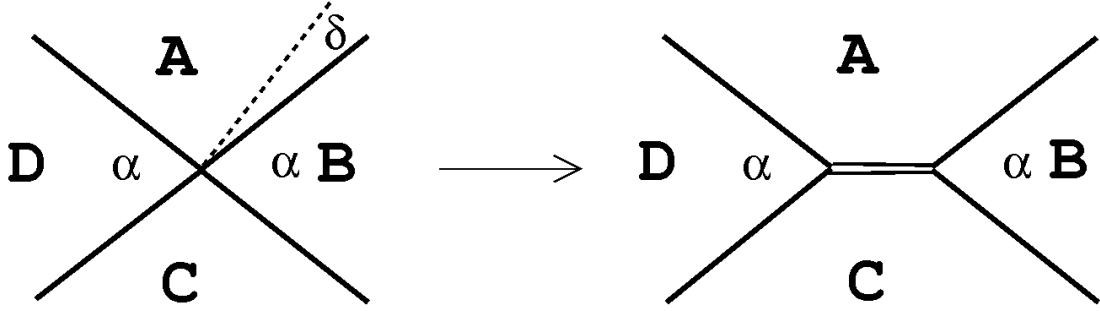


Figure 1: Transition from X junction to neighbouring pair of Y junctions, for wall between vacuum domains labelled A, B, C, D.

3. Criterion for stability of an X type junction

In a model of the kind considered in the previous section, an ordinary X type junction between four vacuum domains A,B,C,D, consists of a string like locus of intersection through which a simple wall of type DC continues as a simple wall of type AB, while a simple wall of type AD continues as a simple wall of type BC. Such an X type junction will always be stable if the model does not admit any compound wall configurations satisfying (2).

The possibility [5] of instability with respect to decomposition into a pair of Y type junctions will however arise if the model is such as to admit compound walls satisfying the condition (2). If the simple walls AB and BC meet at an acute angle α , the decomposition in question would create a Y junction joining them to a compound wall of type AC, while – as shown in Fig. 1 – at the other end of the double wall there would be another Y type junction on which the simple walls AD and DC would meet at the same acute angle α . The original X type junction will be unstable if and only if the double wall of type AC is too weak to prevent the separation of the two Y junctions from increasing, that is to say if and only if

$$T_{\text{II}} < 2T_{\text{I}} \cos \{\alpha/2\}, \quad (3)$$

a condition that is evidently stronger than (2). This is expressible the other way round as the condition that the X junction will be stable if and only if the acute angle α is such that

$$\cos \{\alpha/2\} < \frac{T_{\text{II}}}{2T_{\text{I}}}. \quad (4)$$

Since the range of possible values of the intersection angle is given by $0 < \alpha \leq \pi/2$, which is equivalent to $1 > \cos \{\alpha/2\} \geq 1/\sqrt{2}$, it follows that the criterion for instability of the X junction for all values of the intersection angle is

$$T_{\text{II}} < \sqrt{2} T_{\text{I}}. \quad (5)$$

4. Scalar field models of forced harmonic type

The toy bosonic field models we have been considering belong to a rather extensive category that is describable as being of forced harmonic type. This means that the independent scalar field components, Φ^i say, are to be considered as coordinates on a manifold characterised by a flat or curved Riemannian (positive definite) metric

$$d\lambda^2 = G_{ij} d\Phi^i d\Phi^j, \quad (6)$$

and by a scalar forcing potential V , in terms of which the relevant Lagrangian density in ordinary spacetime, with pseudo Riemannian metric $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ takes the standard form

$$\mathcal{L} = -\frac{1}{2} G_{ij} (\nabla^\nu \Phi^i) \nabla_\nu \Phi^j - V. \quad (7)$$

Generalised non-linear sigma models (of the kind whose properties were used by Bunting [13] for his proof of black hole uniqueness) are included as the subcategory for which the potential function V is set to zero. Scalar field models of the ordinary linear and non-linear kinds that are more commonly considered [14] in physics are included as the subcategory for which the forcing potential is a variable function, $V\{\Phi\}$, but for which the kinetic metric (6) on the space of field values is flat, so in such a kinetically linear model it will be possible to scale the field variables in such a way that the metric components will be given simply by the Kronecker unit matrix as $G_{ij} = \delta_{ij}$.

In a flat static domain wall state of the kind considered in the preceding section, for which the fields depend only on the single cartesian space coordinate x , the surface energy tension T will be the same as the integral along the x axis of the energy density, as given in the limit $\ell \rightarrow \infty$ by the prescription

$$T = U\{\infty\}, \quad (8)$$

where

$$U\{\ell\} = \int_{-\ell/2}^{\ell/2} \left(\frac{1}{2} G_{ij} \frac{d\Phi^i}{dx} \frac{d\Phi^j}{dx} + V \right) dx, \quad (9)$$

in which (to get a finite result) it is to be understood that the potential V has been adjusted if necessary, by the addition of a constant which will have no effect on the field equations obtained from (7), in such a way as to arrange for its minimum (vacuum) value to be zero, $V = 0$.

Stable wall equilibrium states are characterised by the condition that the parametrised field manifold trajectory (as specified by the functions $\Phi^i\{x\}$) with endpoint at the relevant pair of vacuum position (where $V\{\Phi\}$ is minimised) should be such that the energy integral (8) should be a minimum.

Using a prime to denote variation with respect to the parameter λ measuring the distance in the field manifold, so that in particular, according to (6), we shall have

$$G_{ij} \Phi^{i'} \Phi^{j'} = 1, \quad (10)$$

it can be seen that the integrand in (9) can be rewritten as

$$dU = U' d\lambda, \quad (11)$$

with

$$\textcolor{red}{U}' = \frac{1}{2x'} + Vx'. \quad (12)$$

in terms of the rate of variation

$$x' = \frac{dx}{d\lambda}, \quad (13)$$

of the space coordinate x with respect to the field manifold distance given by (6). For given field values at the extremities of some finite range $-\ell/2 < x < \ell/2$, a corresponding equilibrium configuration will be characterised by minimisation of the energy integral (8) with the integrand given by

$$U\{\ell\} = \int \textcolor{red}{U}' d\lambda, \quad (14)$$

subject to the associated constraint

$$\int x' d\lambda = \ell. \quad (15)$$

In terms of an appropriately adjusted value of a Lagrange multiplier P_\perp say, this condition will be equivalent to unrestricted minimisation of the corresponding enthalpy combination

$$H\{\ell\} = U\{\ell\} + P_\perp \ell \quad (16)$$

which will be given by

$$H = \int \left(\frac{1}{2x'} + (V + P_\perp)x' \right) d\lambda. \quad (17)$$

By considering the way this depends on x' for a fixed dependence of the fields on the parameter λ , it can immediately be seen to be necessary for minimisation that the variation rate x' should satisfy the relation

$$x'^2 = \frac{1}{2(V + P_\perp)}, \quad (18)$$

in which P_\perp is interpretable as a constant of integration whose value can be seen to be that of the pressure in the direction orthogonal to the plane of the wall.

The condition (18) can be used to eliminate the involvement of the ordinary space coordinate x , and to express the quantity that (for fixed field values at the endpoints of integration) has to be minimised in the form

$$H = \int \sqrt{2(V + P_\perp)} d\lambda, \quad (19)$$

in which only field manifold variables are involved, and in which the fixed parameters ℓ and P_\perp are not independent but according to (18) must be related by the consistency condition

$$\ell = \int \frac{d\lambda}{\sqrt{2(V + P_\perp)}}. \quad (20)$$

It can be seen from this that when the orthogonal pressure is varied the corresponding variations of $\textcolor{red}{U}$ and ℓ will be related by the formula

$$\frac{d\textcolor{red}{U}}{d\ell} = -P_\perp \quad (21)$$

whereby the interpretation of P_\perp as the relevant orthogonal pressure is made obvious.

In cases such as those of the compact, topologically non-trivial, field manifolds to be considered in the next two sections, a solution for a finite value of ℓ may be extensible over the complete range of x as a “lasagne” type periodic configuration of the kind whose (linear or non-linear) superposition will constitute the kind of cosmological lattice whose investigation is the ultimate motivation for this work. More specifically, the slab thickness ℓ will then be identifiable with the wavelength of the periodicity, so that the corresponding longitudinal wavenumber density will be

$$\nu_\perp = \frac{1}{\ell} = \frac{dP_\perp}{dH}, \quad (22)$$

provided the endpoints of the integration occur at successive maxima or successive minima of the potential as function of x . Thus in particular, for $P_\perp > 0$, the distance ℓ will be the same as the wavelength if the endpoints occur at successive vacuum states.

In such an effectively one dimensional lasagne type configuration, the condition for stability of the large scale averaged system against development of a bunching mode is the positivity of the square v_\perp^2 of the orthogonal propagation speed, of perturbations of the mean density $U\nu_\perp$ – on length scales large compared with the wavelength ℓ – as given by

$$v_\perp^2 = \frac{dP_\perp}{d(U\nu_\perp)} = -\frac{\ell}{H} \frac{dH}{d\ell}. \quad (23)$$

That the requirement $v_\perp^2 > 0$ will indeed be satisfied is shown by the expression

$$v_\perp^{-2} = -\frac{H}{\ell^2} \frac{d\ell}{dP_\perp}, \quad -\frac{d\ell}{dP_\perp} = \int \left(2(V + P_\perp)\right)^{-3/2} d\lambda. \quad (24)$$

It can be seen (using the Schwarz inequality) that the velocity given by (24) can never exceed unity, meaning the speed of light. It will however approach this upper bound when P_\perp becomes very large, so that the kinetic energy becomes much greater than the potential energy (whose particular form will then be irrelevant) which is what happens in the short wavelength limit $\ell \rightarrow 0$, for which one will have

$$U \sim P_\perp \ell, \quad P_\perp \sim \frac{1}{2}(\lambda/\ell)^2, \quad v_\perp^2 \sim 1, \quad (25)$$

where λ is the integrated distance along the field space trajectory from one vacuum state to the next, as measured with respect to the kinetic metric (6)

Our main concern here is not with the short wavelength limit, but on the contrary with isolated wall configurations, which are obtained when the range of integration is unlimited, $\ell \rightarrow \infty$. This large separation condition requires that the integral (20) should be divergent at the extremities of the trajectory in field space, namely the vacuum states characterising the domains in question, where the potential V reaches its minimum. Since, for applicability of the tension formula (8), it is to be understood that (by subtraction, if necessary, of any contribution from a cosmological constant that may be present) this minimum value of V has been adjusted to zero, it can be seen that to obtain an isolated wall configuration the relevant constant of integration must also be taken to be zero,

$$P_\perp = 0. \quad (26)$$

According to (12) this simply gives

$$U' = \sqrt{2V}, \quad (27)$$

and allows one to take the limit $\ell \rightarrow \infty$ in (19) to obtain an expression of the simple form

$$U\{\infty\} = \int \sqrt{2V} d\lambda, \quad (28)$$

for the surface energy to be minimised in order to obtain the equilibrium configuration of the isolated wall.

It can be seen from the formula (27) that this required surface energy density function U is interpretable as a generalisation to the multiscalar case of what has been referred to in the context of a singlet field [15] as a superpotential, and that it is specifiable as the measure of the relevant distance in the field manifold, as evaluated, not with respect to the original kinetic metric G_{ij} , but with respect to a conformally modified field energy metric that is given by

$$dU^2 = \tilde{G}_{ij} d\Phi^i d\Phi^j, \quad (29)$$

with

$$\tilde{G}_{ij} = 2VG_{ij}. \quad (30)$$

The geodesic distance (between the relevant vacua) obtained by minimising the integral $U\{\infty\}$ given by (28) will thus be directly identifiable with the required wall tension T as given by (8). The corresponding distribution of the fields as a function of the orthogonal distance x is given – according to (18) – by a metric that is related to the kinetic metric by a conformal factor that is exactly the inverse of the one that gives the energy metric:

$$dx^2 = \frac{d\lambda^2}{2V} = \frac{dU^2}{4V^2}. \quad (31)$$

5. Analytically integrable cases of pentavac and monovac doublet models

As an illustration of the kind of model that can provide a lattice with stable X-type junctions, I proposed as an interesting prototype example the particular case of what may be concisely referred to as the pentavac model [1]. This is a model with five equivalent vacuum states in a toroidal field space, in which the relevant field variables are a pair of phase variables ϕ and ψ with period 2π with flat kinetic metric given in terms of some fixed mass scale, η say, by

$$d\lambda^2 = \eta^2(d\theta^2 + d\chi^2) = 5\eta^2(d\phi^2 + d\psi^2), \quad (32)$$

and potential V given in terms of some fixed maximum value V_\star by

$$V = \frac{V_\star}{4}(\cos\theta + \cos\chi + 2). \quad (33)$$

in which

$$\theta = 2\phi + \psi, \quad \chi = 2\psi - \phi. \quad (34)$$

A model of this topologically non-trivial kind is obtainable [1] as a low energy limit $\varepsilon \rightarrow 0$ from a topologically simple extended model of with broken $U(1) \times U(1)$ symmetry, of the kind is decribed in the appendix.

According to (30) the energy metric obtained from (32) and (33) on the toroidal space of phase variables ϕ and ψ will be given by the formula

$$dU^2 = 2\eta^2 V (d\theta^2 + d\chi^2), \quad (35)$$

in which it is useful to rewrite the formula (33) for the relevant potential as

$$V = V_\star \left(\frac{1}{2} \cos^2\{\theta/2\} + \frac{1}{2} \cos^2\{\chi/2\} \right). \quad (36)$$

For such a potential, the corresponding geodesic Hamilton Jacobi equation, namely

$$\frac{1}{2\eta^2 V} \left(\left(\frac{\partial SS}{\partial \theta} \right)^2 + \left(\frac{\partial SS}{\partial \chi} \right)^2 \right) = 1, \quad (37)$$

turns out to have the convenient property of being separable with respect to the variables θ and χ . By taking the Jacobi action variable SS to be a sum,

$$SS = SS_\theta + SS_\chi, \quad (38)$$

of single variable functions, the equation (37) can be seen to be reducible to the form

$$\left(\frac{dSS_\theta}{d\theta} \right)^2 - \eta^2 V_\star \cos^2\{\theta/2\} = \eta^2 V_\star \cos^2\{\chi/2\} - \left(\frac{dSS_\chi}{d\chi} \right)^2, \quad (39)$$

in which the terms on the left depend only on θ while those on the right depend only on χ , which means that both sides must be equal to a constant of integration, \mathcal{K} say. This means that the corresponding generalised momentum variables, which are specifiable according to (35) as

$$\mathcal{P}_\theta = 2\eta^2 V \frac{d\theta}{dU}, \quad \mathcal{P}_\chi = 2\eta^2 V \frac{d\chi}{dU}, \quad (40)$$

will satisfy equations of the form

$$\mathcal{P}_\theta^2 = \eta^2 V_\star \cos^2\{\theta/2\} + \mathcal{K}, \quad \mathcal{P}_\chi^2 = \eta^2 V_\star \cos^2\{\chi/2\} - \mathcal{K}. \quad (41)$$

It is to be recalled that the only trajectories to which this derivation is applicable are those that begin and end on vacuum states, namely the field values for which $\cos^2\{\theta/2\} = \cos^2\{\chi/2\} = 0$. In order for the momentum values given by (41) to remain real at these end points it is evident that neither \mathcal{K} nor $-\mathcal{K}$ can be negative, which is only possible if the constant itself vanishes,

$$\mathcal{K} = 0. \quad (42)$$

It can thereby be concluded that the equations of motion for the relevant geodesics are given simply by

$$4\eta^2 V^2 \left(\frac{d\theta}{dU} \right)^2 = V_\star \cos^2\{\theta/2\}, \quad 4\eta^2 V^2 \left(\frac{d\chi}{dU} \right)^2 = V_\star \cos^2\{\chi/2\}. \quad (43)$$

It can be seen from this that, independently of parametrisation, the trajectory in phase space will be obtainable by integrating

$$\frac{d\theta}{\cos\{\theta/2\}} = \pm \frac{d\chi}{\cos\{\chi/2\}}, \quad (44)$$

and that the corresponding expression for the energy variation (in the “right” direction) will be given by

$$dU = 2\eta \sqrt{V_\star} (\cos\{\theta/2\} d\theta \pm \cos\{\chi/2\} d\chi). \quad (45)$$

An example of a simple wall trajectory (such as the straight line AB in Fig. 2) is obtainable by holding χ fixed at a value such that $\cos\{\chi/2\} = 0$ (so that the ratio on the right of (44) is indeterminate) and letting θ vary from $-\pi$ to π . According to (31) the wall profile of the field θ as a function of the orthogonal space coordinate x will be given by the relation

$$\cot \left\{ \frac{\theta - \pi}{2} \right\} = \sinh \left\{ \frac{x}{\delta_\star} \right\}. \quad (46)$$

in which the effective wall thickness scale δ_\star is determined by the relevant mass scale m_\star as

$$\delta_\star = \frac{2\eta}{m_\star^2}, \quad m_\star^4 = V_\star. \quad (47)$$

In this case there will be no contribution from the right of (44), so the complete integral $U\{\infty\}$ giving the wall tension T_I works out as

$$T_I = 4 m_\star^2 \eta = 2 V_\star \delta_\star. \quad (48)$$

To get a compound wall trajectory (of the kind illustrated by the curves AC in Fig. 2) the variable χ is no longer held fixed but is also allowed, like θ to vary from $-\pi$ to π . Such a trajectory is obtainable by choosing $\pm = +$ in equation (44), which is then easily integrable to give

$$\cot \left\{ \frac{\chi - \pi}{4} \right\} = \kappa \cot \left\{ \frac{\theta - \pi}{4} \right\}, \quad (49)$$

where κ is a dimensionless constant of integration. It takes the value $\kappa = 1$ in the trivial case of the straight diagonal trajectory given by the equation $\chi = \theta$. For any non-zero value of κ , the separate energy contributions from the pair of terms on the right of (45) will integrate to

the same final result. Thus the various trajectories given by different values of κ in (49) are energetically degenerate, all giving the same compound wall energy

$$T_{\text{II}} = 4V_{\star} \delta_{\star}. \quad (50)$$

The condition of being just twice the simple wall energy is an obvious consequence of the fact that although their topology is globally interwoven, the separate field combinations θ and χ behave locally as a pair of decoupled scalars, so their wall configurations can travel through each other without interaction.

According to the analysis in the preceding sections, this feature

$$T_{\text{II}}/2T_{\text{I}} = 2, \quad (51)$$

is just what is marginally needed to ensure stability (against disintegration into pairs of Y junctions) of the X-junctions, whatever their crossing angle α may be (not just when it is near a right angle as would be needed in models providing a lower value of $T_{\text{II}}/T_{\text{I}}$.) Without going into these quantitative details, the stability, as previously claimed [1], of the X-junctions in this particular case was heuristically obvious in advance from the lack of any mechanism of disintegration (into pairs of Y junctions) in view of the effective absence of interaction between the separate θ and χ fields that respectively characterise the intersecting walls. (For sufficiently small but non-zero values of the parameter ε in the corresponding topologically simple extended model [1] described in the appendix, continuity implies that X-junction stability will still hold except for correspondingly small – meaning highly acute – values of the intersection angle α .)

Although there is no interaction between walls produced by the separate variation of θ and χ , it is important to notice that there will be an interaction between the parallel walls in the periodic solution produced by the variation of a single one of the separate variables, θ say, for a finite value of the wavelength ℓ as given by the formula (20) for the separation distance. In such a lasagne type configuration there will be an interaction, interpretable as an effect of mutual repulsion, due to the build up, as ℓ decreases, of the corresponding orthogonal pressure P_{\perp} . In the separable case under consideration, it can be seen from (20) and (19) that, in terms of the “first” and “second” kinds of elliptic integral,

$$K\{\mu\} = \int_0^1 \frac{dt}{\sqrt{(1-t^2)(1-\mu t^2)}}, \quad E\{\mu\} = \int_0^1 \sqrt{\frac{1-t^2}{1-\mu t^2}} dt, \quad (52)$$

the relations between pressure, separation, and the enthalpy density H will be given by

$$\frac{\ell}{2\delta_{\star}} = \sqrt{\mu} K\{\mu\}, \quad \frac{H}{T_{\text{I}}} = \frac{E\{\mu\}}{\sqrt{\mu}}, \quad \mu = \frac{V_{\star}}{V_{\star} + 2P_{\perp}}, \quad (53)$$

where δ_{\star} is the effective wall thickness as given by (47). It follows that in the long wavelength limit $\ell \rightarrow \infty$, the relevant pressure will be given asymptotically by

$$P_{\perp} \sim 8V_{\star} e^{-\ell/\delta_{\star}}, \quad (54)$$

while in terms of the tension T_I of an isolated wall, as given by (48), it can be seen from (21) that the surface energy density per period will be given by the asymptotic formula

$$U \simeq T_I \left(1 + 4e^{-\ell/\delta_\star}\right). \quad (55)$$

According to (23) the velocity of large lengthscale longitudinal perturbations will be given, in this long wavelength limit $\ell \gg \delta_\star$, by an expression of the corresponding form

$$v_\perp \sim \frac{2\ell}{\delta_\star} e^{-\ell/2\delta_\star}, \quad (56)$$

whose reality is what guarantees stability against bunching.

6. Beta monovac winding models

The fields ϕ and ψ in the pentavac model on the preceeding section were set up as the respective phases of unimodular complex field variables $e^{i\phi}$ and $e^{i\psi}$ that could themselves be considered to have been obtained by imposition of a low energy restraint on corresponding complex variables Φ and Ψ whose modulus at much higher energy would no longer be restrained. A new model that would be physically indistinguishable in the low energy limit characterised by the unimodularity condition could be set up by instead taking the independent fields to be the unimodular complex variables $e^{i\theta}$ and $e^{i\chi}$ that are defined by the combinations θ and χ specified by (34)

Whereas the toroidal space occupied by the original pair of unimodular field variables $e^{i\phi}$ and $e^{i\psi}$ can be covered by any one of the four large square patches bounded by dotted lines in Fig. 2, the (much smaller) toroidal space occupied by the new pair of unimodular field variables $e^{i\theta}$ and $e^{i\chi}$ can be covered just by the small square with vertices at the positions marked A,B,C,D, which in the new model are to be identified, so that there will now be just a single vacuum state instead of five. In the new “monovac” model (a doublet generalisation of the singlet sine Gordon equation) that is obtained in this way, a simple wall of the kind exemplified by the trajectory AB will still be a topologically stable membrane defect, but of the kind known specifically as a “winding”, to distinguish it from the more commonly discussed kind of “open kink” for which (as in the pentavac case) the vacuum states on either side are of distinct varieties.

Both the original pentavac model and the new monovac model can be regarded as special separable cases within larger families of respectively pentavac and monovac models for which the potential is given in terms of a fixed index $\beta > 0$ by an expression of the form

$$V = \frac{m_\star^4}{2} \left(\cos^2\{\theta/2\} + \cos^2\{\chi/2\} \right)^\beta, \quad (57)$$

which includes the special separable example (36) as the particular case for which $\beta = 1$.

It is apparent that – for sufficiently large values of the angle δ of deviation from orthogonality in Fig. 1 – the possibility of instability of an X type junction with respect to disintegration into Y type junctions [5] will occur in such (monovac or pentavac) models if and only if the strict inequality $\beta < 1$ is satisfied. By comparing the double wall trajectory parametrised by the symmetric relation $\chi = \theta$ with the simple wall trajectory given by the fixed value $\chi = \pi$

it can be seen from (57) that the energy T_{II} of the former (as represented by the diagonal AC in Fig. 2) will be related to the energy T_{I} of the latter (as represented by AB) by

$$T_{\text{II}} = (\sqrt{2})^{1+\beta} T_{\text{I}}. \quad (58)$$

The actual value of the simple wall tension in such a case can be seen to be given in terms of the thickness parameter δ_* defined by (47), and an order of unity factor I_β , by

$$T_{\text{I}} = 4 m_*^2 \eta I_\beta, \quad I_\beta = \int_0^1 \frac{y^\beta dy}{\sqrt{1-y^2}}, \quad (59)$$

which in the particularly interesting case $\beta = 2$ gives $T_{\text{I}} = \pi m_*^2 \eta$.

It evidently follows from (58) that for $\beta > 1$ the double wall configuration (such as AC) will be unstable with respect to decomposition into a pair of simple walls (such as AB and BC) between which there will be an effective repulsion. On the other hand, for $\beta < 1$ a complementary pair of simple walls (such as AB and BC) would be attractive, and in such a case, according to (4) an X junction will be unstable if but only if the deviation angle δ in (1) is large enough to satisfy the strict inequality

$$\sin \delta > 2^\beta - 1, \quad (60)$$

a condition that would evidently be impossible for $\beta \geq 1$.

The foregoing considerations remain true when – to be cosmologically realistic – the univac doublet model is extended to a univac triplet model by the inclusion of a third unimodular scalar field, $e^{i\sigma}$ say, acting in the same way as the others, so that the complete kinetic metric will be given in terms of a relatively large mass scale η by

$$d\lambda^2 = \eta^2 (d\theta^2 + d\chi^2 + d\sigma^2), \quad (61)$$

while in terms of a smaller mass scale m_* the complete potential function will take the form

$$V = \frac{m_*^4}{2} \left(\cos^2\{\theta/2\} + \cos^2\{\chi/2\} + \cos^2\{\sigma/2\} \right)^\beta, \quad (62)$$

which reduces to (57) wherever $\cos\{\sigma/2\} = 0$.

By a mutually orthogonal superposition of the lasagne type configurations corresponding to variation of θ , χ and σ respectively, it can be seen that for $\beta > 1$ such a model can provide a robustly stable cubical (rather than merely square) lattice of the kind needed for the averaged stress tensor to be isotropic. For a single one of these lasagne configurations, corresponding just to the variation of θ say, with the other two variables fixed at the energy minimising values $\cos^2\{\chi/2\} = \cos^2\{\sigma/2\} = 0$, it can be seen from (20) that for $\beta > 1$ – instead of falling off in the exponential manner described by (54) – the dependence on the separation distance ℓ of the orthogonal pressure will be given asymptotically, as $\ell \rightarrow \infty$, by a power law of the form

$$P_\perp \sim \frac{m_*^4}{2} \left(\frac{\delta_\beta}{\ell} \right)^{2\beta/(\beta-1)}, \quad (63)$$

in which δ_β is of the same order of magnitude as the original thickness scale δ_* defined by (47), to which it is related by the specification

$$\delta_\beta = J_\beta \delta_*, \quad J_\beta = \int_0^\infty \frac{dz}{\sqrt{z^{2\beta} + 1}}, \quad (64)$$

which, in the quartic case $\beta = 2$ provides the expression $J_2 = 4 \Gamma^2\{5/4\}/\sqrt{\pi} = \varsigma\pi/2$ in terms of a numerical factor close to unity given by $\varsigma \simeq 1.18$.

It follows that the analogue for $\beta > 1$ of the asymptotic formula (55) for the surface energy density of each slab of thickness ℓ will be given by

$$U \simeq T_I + \frac{\beta - 1}{\beta + 1} \ell P_\perp, \quad (65)$$

and that according to (23) the analogue of (56) for the squared velocity of large lengthscale longitudinal perturbations takes the form

$$v_\perp^2 \sim \frac{\beta I_\beta}{2(\beta - 1)J_\beta} \left(\frac{\delta_\beta}{\ell} \right)^{(\beta+1)/(\beta-1)}, \quad (66)$$

which is manifestly positive, as required for stability, whenever $\beta > 1$.

7. Macroscopic comportment

One of the objections raised against the idea of attributing the cosmological acceleration to a regular domain wall lattice was the lack of a plausible mechanism to prevent parallel walls from drifting together in the long run and eventually undergoing mutual annihilation. It is therefore to be emphasised that this is not a danger in the kind of monovac model advocated here, in which the parallel wall number density is interpretable as a conserved topological winding number, and the parallel walls are protected against getting too close by the effect whereby, when the separation distance ℓ gets too small, the orthogonal pressure P_\perp in (20) will cease to be negligibly small, and will build up so as to produce a mutually repulsive force.

The simplest cosmologically viable category is that of the separable – $\beta = 1$ – kind of monovac triplet model (effectively a superposition of three independent sine-Gordon singlets) which provides a cubic lattice with a cosmologically comoving lengthscale ℓ and hence cell number density $n = 1/\ell^3$ for which the relations (53) implicitly provide a corresponding equation of state whereby the cosmological energy density ρ , will be given by

$$\rho = 3 \frac{U}{\ell}, \quad U = H - P_\perp \ell, \quad (67)$$

while the corresponding isotropic cosmological pressure P , which will be given simply by

$$P = P_\perp - 2 \frac{U}{\ell} = 3P_\perp - 2 \frac{H}{\ell}, \quad (68)$$

which means that the cosmologically important ratio $w = P/\rho$ will be given by the formula

$$w = -\frac{2}{3} + \frac{P_\perp}{\rho}. \quad (69)$$

At a much earlier epoch, when the winding wavelength ℓ would have been comparable with or even much less than the wall thickness scale δ_\star given by (47), it can be seen from (25) that one would have had

$$w \simeq -\frac{1}{3} \quad \text{with} \quad \frac{P_\perp}{\rho} \sim \frac{1}{3}, \quad \rho \sim \frac{6\pi^2 \eta^2}{\ell^2}. \quad (70)$$

However at the present epoch it is to be presumed that the (cosmologically comoving) length-scale ℓ will have become very much larger, $\ell \gg \delta_\star$, and hence that it will be possible to neglect the final term in (69), which would fall off rapidly with a negative power law dependence on ℓ for $\beta > 1$, and with an exponential dependence given by the asymptotic formula

$$\frac{P_\perp}{\rho} \sim \frac{4\delta_\star}{3\ell} e^{-\ell/\delta_\star}, \quad (71)$$

for the case $\beta = 1$, so that (in all these cases) one would ultimately attain the observationally admissible [16] value given, using (59), by

$$w \simeq -\frac{2}{3}, \quad \text{with} \quad \rho \approx 10 \frac{m_\star^2 \eta}{\ell}. \quad (72)$$

For a simple perfect fluid such an equation of state would of course be unacceptably unstable. The mechanism for the stabilisation confirmed in the present work is of exactly the kind that is describable by treating the large scale averaged system as a perfectly elastic solid (in which the three relevant fields θ , χ , σ would be comoving “base space” coordinates) of the kind originally envisaged by Bucher and Spergel[6], of which it is thus a prefect example.

In order to have become dominant, the density (72) must have recently reached the order of magnitude of the baryonic mass density which is given in terms of the proton mass m_p and the cosmological temperature Θ by $\rho_b \approx 10^{-8} m_p \Theta^2$. Since the temperature has a contemporary value given in terms of the electron mass m_e by $\Theta_c \approx 10^{-9} m_e$, it can be seen to follow that the ratio of the contemporary value ℓ_c the mesh spacing ℓ to the value (in the millimeter range) of the contemporary thermal wavelength $\approx \Theta_c^{-1}$ must be given by

$$\Theta_c \ell_c \approx 10^{25} \frac{\eta}{m_p} \left(\frac{m_\star}{m_e} \right)^2. \quad (73)$$

According to the line of reasoning in the preceding analysis [1], the Kibble type wall formation mechanism that was envisaged would be likely to provide values of this length ratio $\Theta_c \ell$ in the range from 10^{11} , which would be obtained for $m_\star \approx 10^{-5} \eta$ with $\eta \approx m_e$, up to about 10^{12} which would be obtained for $m_\star \approx 10^{-2} \eta$ with $\eta \approx 10^{-2} m_e$. These values are comparatively small (not comparable, as was stated due to a transcription error) with respect to the interstellar distance scale, of the order of several parsecs, which is what would be obtained for ℓ_c if one used the rather larger mass values $m_\star \approx \eta \approx 10^{-1} m_e$ that were suggested by an analysis of the cruder kind used in earlier work [7]. However it now seems

that all such reasoning, whether in its earlier form or in the more refined [1] version, should be considered to be effectively obsolete, because its generic outcome will be the formation of disorganised lattice configurations that would be insufficiently stable, unlike the superposed lasagne type configurations envisaged here.

The whole question of the wall formation process needs to be entirely reconsidered in the context of the stabilisation mechanism considered here, which does not work for random wall lattice configurations, but is essentially dependent on the existence of effectively conserved winding numbers with sufficiently large values, representing the effect of systematic winding in the same sense over a cosmologically large scale. Even for an underlying field model of the appropriate kind (as illustrated by the examples considered above) the ordinary kind of Kibble mechanism considered so far [7, 1] would give rise to random combinations of positive and negative windings, corresponding to what might be called walls and anti-walls, which in the long run would be unstable with respect to mutual annihilation. The work in the following section suggests, however, that it be possible to overcome this problem by incorporating the effect of the kind inflationary process [17] that has been developed for dealing with what is known as the horizon problem. On the basis of this revised analysis it will be found that the most plausible mean mass values are not smaller but larger than those that were originally envisaged, with the implication that the present mesh lengthscale ℓ_c should actually be expected to be of intergalactic order.

8. An inflationary mechanism?

The horizon problem was posed in the context of traditional cosmology by the need to account for the approximate homogeneity – indicative of causal contact in the past – that is observed all the way out to the present value of the Hubble radius R as defined, in terms of the proper time derivative \dot{a} of a comoving lengthscale a say, by $1/R = \dot{a}/a$. This suggests that the radius, ξ say, of effective causal correlation must be larger than this, $\xi \gtrsim R$. The problem is that in the traditional scenario of a decelerating radiation dominated universe this Hubble radius has the same order of magnitude as the particle horizon that provides an extreme upper limit on ξ . To get over this, the idea put forward by Guth [17] was that such a restriction would no longer apply if, at some stage in the past, there had been a sufficiently long period of inflation during which the universe was subject to acceleration, $\ddot{a} > 0$ of the kind that seems to have recently started again.

When such considerations were originally introduced by Guth [17], the intended application was to the breakdown of grand unification leaving a relic distribution of monopole particles whose density per Hubble volume was assumed to have been at least of the order of unity at the relevant initial time t_i , implying an initial number density $n_i \gtrsim \xi_i^{-3}$. The monopole problem was actually even worse than commonly supposed since such a lower limit can plausibly be strengthened by taking account of the consideration that thermal fluctuations corresponding to the temperature Θ_i (in Planck units) at that epoch might have initially produced roughly of the order of one of the relevant particles or antiparticles per thermal volume Θ^{-3} , of which there would have then have been $N \approx (\xi_i \Theta_i)^3$ in the correlated volume under consideration. As the correlation radius increased, the created number in the corresponding volume would

also increase by a factor $(\xi/\xi_i)^3(a/a_i)^{-3}$ to give

$$N \approx (\xi \Theta_i a_i / a)^3. \quad (74)$$

Due to their causal interaction, these particles and antiparticles would however have undergone mutual annihilation, except for a relatively small excess with the same sign (e.g. with no antiparticles) whose number would have had a root mean square value that can be estimated by a random walk argument to be of the order of \sqrt{N} , as given by the estimate

$$\sqrt{N} \approx (\xi \Theta / Z_i)^{3/2}. \quad (75)$$

in which the thermal evolution formula

$$\Theta / \Theta_i = Z_i a_i / a, \quad (76)$$

has been used to define a reheating factor Z_i factor of the kind that was supposed by Guth to have become extremely large, and that for consistency with the second law of thermodynamics must at least increase monotonically from an initial value $Z_{ii} = 1$. In terms of the amplification factor

$$\aleph = \xi / R, \quad (77)$$

this means that the order of magnitude of the number density with which they would have ultimately emerged would be expressible as

$$n \approx \left(\frac{\Theta}{\aleph Z_i R} \right)^{3/2}. \quad (78)$$

Since the ratio of the present cosmological radius $R_c \sim 10^{59}$ to the thermal lengthscale given by the present cosmological temperature $\Theta_c \approx 10^{-31}$ is given roughly by $R_c \Theta_c \approx 10^{28}$, the implication is that the number of such particles in a Hubble volume would be given now by

$$n_c R_c^3 \approx \left(\frac{R_c \Theta_c}{\aleph_c Z_{ic}} \right)^{3/2} \lesssim 10^{42}, \quad (79)$$

in which the lower limit on the right is obtained by postulating the order of unity values that are the smallest conceivable possibilities for the inflation amplitude \aleph_c and the reheating factor Z_{ic} at the present epoch. Guth's idea [17] for the latter, in the context of the monopoles predicted by the Grand unification theory (which was more fashionable then than now) was to postulate an enormous value $Z_{ic} \sim 10^{28}$ in order to get the right hand side of (79) down to near the order of unity.

At the opposite extreme, in the context of the baryon formation problem, it is interesting to note that although it is rather large, the maximum value on the right of (79) is barely the square root of the value – nearer 10^{76} – of Dirac's cosmological baryon number, thus falling far short of what would be needed even to make a single star, so it can not be hoped that a mechanism of this kind would suffice to solve the problem of the excess of baryons over antibaryons.

What such a mechanism might conceivably do, however, would be to solve the analogous problem of getting the excess of positive over negative windings that would be needed to provide the kind of domain wall lattice discussed above, in which the lattice spacing ℓ would correspond to a cell number density $n = \ell^{-3}$, so that the formula (78) would provide for the the relevant contemporary values the analogous estimate

$$\frac{\Theta_c \ell_c}{\sqrt{\aleph_c Z_{ic}}} \approx \sqrt{R_c \Theta_c} \approx 10^{14}, \quad (80)$$

which means that, even for very moderate values of the ratios \aleph_c and Z_{ic} , the contemporary value ℓ_c of the wall spacing now would at least be large compared with the size of the solar system. To avoid having $\ell_c \gtrsim R_c$, which would mean that the walls would be entirely inflated away (in the manner envisaged by Guth for the monopoles of grand unification) it can be seen that the inflation factor must satisfy

$$\sqrt{\aleph_c Z_{ic}} \ll 10^{14}. \quad (81)$$

Comparing (80) with the matching condition (73) one sees that it requires that the relevant mass ratios should be related by

$$\frac{\eta}{m_e} \left(\frac{m_\star}{m_e} \right)^2 \approx 10^{-8} \sqrt{\aleph_c Z_{ic}}. \quad (82)$$

On the assumption that the potential mass scale is small compared with the kinetic mass scale, meaning $m_\star \ll \eta$, it can thereby be seen to follow from (81) that the lighter mass scale must be subject to a rather severe upper mass limit given by

$$m_\star \ll 10^2 m_e. \quad (83)$$

An opposing restriction comes from the astrophysical consideration that, to avoid undermining the experimentally well confirmed scenario of chemical element formation in the temperature regime $\Theta \lesssim m_e$, the postulated inflation following the relevant phase transition, with $\Theta_i \lesssim \eta$, must have been finished before this stage, which evidently imposes the requirement that the heavier mass scale should be subject to a lower mass limit given by

$$\eta \gg m_e. \quad (84)$$

9. Conclusions

To compute the stability of an X type junction in the framework of a particular field theoretical model it is not necessary to work with the essentially two dimensional geometry that is actually involved: it will suffice to consider the effectively one dimensional problems of plane wall configurations of simple and double type, in order to obtain the corresponding

energy densities T_I and T_{II} whose ratio is needed. A right angle junction ($\alpha = \pi/2$ in Fig. 1) will always be stable so long as

$$T_{II}/T_I > \sqrt{2}, \quad (85)$$

but a larger ratio will be needed for stability of X junctions at other angles.

For models of the forced harmonic kind it has been shown that the required tensions are obtainable as the field space distances between the relevant vacua, as measured with respect to the energy metric given by (30). For the separable models such as the pentavac doublet model [1] that was subject to question [5], this geodesic method has been used for the explicit evaluation of the relevant tensions, and it has been shown that the stability condition is indeed satisfied. Finally attention has been drawn to a cosmologically more promising class of monovac triplet models for which the stability condition is guaranteed whenever the relevant index β exceeds unity.

At a macroscopic level, the particular models considered here are well described by the formalism of an elastic solid medium, but it is to be observed that, convenient though it is [6], this feature is not essential for their stability. The winding stabilisation mechanism for the superposed lasagne type configurations envisaged here would work just as well if, instead of three (the minimum compatible with an isotropic total stress tensor) a higher number of independent scalar fields were invoked. In that case one would obtain more than triply superposed lasagne type configurations which would be just as stable, but in which there would be too many degrees of freedom for the system to behave as a simple elastic (or even hyperelastic [18]) solid unless coupled with something else that ensured the required cohesion. The lack of a plausible cohesion mechanism was one of the weak points in the kinds of solid lattice scenario that were originally considered. It is therefore to be emphasised that this drawback does not apply to the stability mechanism considered here, for which no such cohesion mechanism is needed.

The drawback that remains is, as remarked in the introduction, the need for the system to have been created in a configuration that is wound up coherently in the same sense (without string winding defects, and with only “positive” walls as opposed to “anti-walls”) over a sufficiently large cosmological scale. This last drawback is serious, but (particularly in view of the likely involvement of an anthropic selection mechanism) it should not be considered to be automatically fatal. It should rather be taken as a challenge that is comparable with (and perhaps related to) the unavoidable challenges posed by the problem of the parity breaking that is associated with the observationally well established preponderance of ordinary matter over anti-matter [11], and by the – rather less intractable and perhaps more directly relevant – horizon problem, which provides a major incentive for the hypothesis of cosmological inflation, and also motivates consideration of effects of multiconnectedness [8].

The provisional investigation in the preceeding section suggests that it may indeed be possible for an appropriate inflation scenario to provide what is needed. An attractive feature of this approach is that while the concomitant restrictions (81), (83), (84) rule out more exotic mass values, they can be satisfied in a very reasonable way by postulating that the relevant potential and kinetic mass scales have the most obvious orders of magnitude, namely

$$m_\star \approx m_e, \quad \eta \approx m_p. \quad (86)$$

This would require that the relevant late inflation stage – initiated with temperature Θ_i at or below the B.e.v. level – should have provided a mean inflation factor $\sqrt{\aleph_c Z_{ic}} \approx 10^{11}$, in which Z_{ic} is the reheating factor and \aleph_c is the factor (if any) by which the relevant range of causal influence exceeds the Hubble radius R_c . The ensuing wall mesh scale would then be

$$\ell_c \approx 10^{-3} R_c, \quad (87)$$

which means that, in a double inflation scenario, a previous such mean inflation factor of 10^3 (due perhaps just to reheating by 10^6) would have sufficed to sweep away any grand unification monopoles. It is noteworthy that, in such a scenario, the contemporary mesh length ℓ_c would be in rather satisfactory agreement with the order of the ten megaparsec scale to which (in the context of double inflation) attention has been drawn [19] as the intergalactic threshold for an enhancement of the power spectrum of the observed structure. This agreement would also be compatible with a more extreme mass parameter ratio and higher inflation temperature, such as might be given, with Θ_i above the T.e.v. level, by $m_\star \approx 10^{-2} m_e$ with $\eta \approx 10^4 m_p$.

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Appendix: construction of reduced field models.

In a universe with appropriate toroidal topology (so as to admit configurations without any string defects) the field models considered above might be postulated to have an essentially fundamental status. However in might be computationally convenient (and in other contexts, such as that of Kibble type mechanisms [9, 10, 11], more natural) to consider such topologically non-trivial field models to be derived as approximate effective models from higher dimensional field spaces with simple geometry, in which the relevant total potential V_{tot} contains a sufficiently steeply variable confining contribution, V_{cof} say, having a degenerate minimum on a submanifold constituting a reduced field space with the non trivial geometry in question.

Provided the residual part V_{red} in the decomposition

$$V_{\text{tot}} = V_{\text{cof}} + V_{\text{red}} \tag{88}$$

is relatively slowly varying in the immediate neighbourhood of this submanifold, its value V_{red} thereon is all that will be needed for determining the effective behaviour of the system whenever the available field energy is insufficient to excite significant deviations from the this submanifold. Thus, in such a low energy limit, the system will be effectively describable just in terms of the reduced model constituted by the reduced field space with the effective potential given by the residual part V_{red} .

The pentavac doublet model considered in Section 5 was originally obtained [1] in just this way by taking the limit $\varepsilon \rightarrow 0$ in a broken $U(1) \times U(1)$ model involving a pair of complex fields with a flat kinetic metric of the standard $O(4)$ invariant form

$$d\lambda^2 = 5\eta^2(d\Phi d\bar{\Phi} + d\Psi d\bar{\Psi}), \quad \Phi = |\Phi|e^{i\phi}, \quad \Psi = |\Psi|e^{i\psi}, \quad (89)$$

and with a potential given as a sum of the form (88) with a $U(1) \times U(1)$ invariant confining part

$$V_{\text{cof}} = \frac{V_{\#}}{4} \left((|\Phi|^2 - 1)^2 + (|\Psi|^2 - 1)^2 + 2\varepsilon^2/(1 - \varepsilon) \right), \quad V_{\#} = m_{\#}^4, \quad (90)$$

(in which the final constant term has been included to adjust the minimum of the total potential to the standard value $V_{\text{tot}} = 0$) together with a weakly variable residual part given in terms of the combinations (34) by

$$V_{\text{red}} = \frac{V_{\star}}{4} \left(|\Phi|^2 |\Psi|^2 (\cos \theta + \cos \chi) + 2 \right), \quad V_{\star} = m_{\star}^4 = \varepsilon V_{\#}. \quad (91)$$

The ratio

$$\varepsilon = \frac{V_{\star}}{V_{\#}} = \left(\frac{m_{\star}}{m_{\#}} \right)^4 \quad (92)$$

will act as a small symmetry breaking parameter. In this case the minimum of the confining potential V_{cof} that will be used as the reduced field potential consists of the toroidal locus where $|\Phi|^2 = |\Psi|^2 = 1$, to whose neighbourhood the field will be closely confined when $m_{\#} \gg m_{\star}$ unless the available field energy density is relatively large compared with V_{\star} . This neighbourhood will include the five vacuum positions where V_{tot} vanishes, which happens wherever $\cos \theta = \cos \chi = -1$ on the nearby toroidal locus where $|\Phi|^2 = |\Psi|^2 = 1/(1 - \varepsilon)$.

The same reduced field model is more elegantly obtainable from a variant in which the final terms in (90) and (91) are altered so as to give a vanishing minimum not just for the sum V_{tot} , but also for the separate contributions V_{cof} and V_{red} , by taking them to be given by the prescriptions

$$V_{\text{cof}} = \frac{m_{\#}^4}{4} \left((|\Phi|^2 - 1)^2 + (|\Psi|^2 - 1)^2 \right), \quad (93)$$

and

$$V_{\text{red}} = \frac{m_{\star}^4}{4} |\Phi|^2 |\Psi|^2 (\cos \theta + \cos \chi + 2) = \frac{m_{\star}^4}{2} |\Phi|^2 |\Psi|^2 \left(\cos^2 \{\theta/2\} + \cos^2 \{\chi/2\} \right), \quad (94)$$

which are such as to ensure that the vacua occur exactly on the locus where $|\Phi|^2 = |\Psi|^2 = 1$ even for finite values of the dimensionless ratio $(m_{\star}/m_{\#})^4$.

A category of monovac triplet models of the kind considered in Section 6 can be similarly obtained as a low energy limit, for $m_* \ll m_\sharp$, from broken $U(1) \times U(1) \times U(1)$ models with a flat kinetic metric of the standard $O(6)$ invariant form

$$d\lambda^2 = \eta^2 (d\Theta d\bar{\Theta} + dX d\bar{X} + d\Sigma d\bar{\Sigma}), \quad (95)$$

for

$$\Theta = |\Theta|e^{i\theta}, \quad X = |X|e^{i\chi}, \quad \Sigma = |\Sigma|e^{i\sigma}, \quad (96)$$

with a confining term of the form

$$V_{\text{cof}} = \frac{m_\sharp^4}{4} \left((|\Theta|^2 - 1)^2 + (|X|^2 - 1)^2 + (|\Sigma|^2 - 1)^2 \right), \quad (97)$$

The required form (62) of the potential V for the ensuing reduced model is then obtained by imposing the unimodular restriction on a weakly variable residual part that is taken to be given in terms of a fixed index β by

$$V_{\text{red}} = \frac{m_\star^4}{2} \left(|\Theta|^2 \cos^2\{\theta/2\} + |X|^2 \cos^2\{\chi/2\} + |\Sigma|^2 \cos^2\{\sigma/2\} \right)^\beta, \quad (98)$$

so that its dependence on the field moduli will be quartic for $\beta = 2$, and quadratic in the separable case, $\beta = 1$.

An intrinsically equivalent reduced model is obtainable from an algebraically simpler alternative, albeit with not just one but 8 distinct vacua, that is specifiable in terms of 6 real variables, $\mathcal{X}_i, \mathcal{Y}_i$, for $i=1, 2, 3$, which combine as a set of 3 complex variables, whose phases determine the corresponding angles $\theta = 2\zeta_1, \chi = 2\zeta_2, \sigma = 2\zeta_3$, in the form

$$\mathcal{X}_i + i\mathcal{Y}_i = \mathcal{Z}_i = |\mathcal{Z}_i|e^{i\zeta_i}, \quad (99)$$

with the potential contributions in (88) given (in a form that is purely quartic for $\beta = 2$) by

$$V_{\text{cof}} = \frac{m_\sharp^4}{4} \sum_i (\mathcal{X}_i^2 + \mathcal{Y}_i^2 - 1)^2, \quad V_{\text{red}} = \frac{m_\star^4}{2} \left(\sum_i \mathcal{X}_i^2 \right)^\beta, \quad (100)$$

so that unimodularity in the low energy limit is obtained in the same way as before on the assumption that $m_\sharp \gg m_\star$. Provided the flat kinetic metric is taken to have the form

$$d\lambda^2 = 4\eta^2 \sum_i (d\mathcal{X}_i^2 + d\mathcal{Y}_i^2), \quad (101)$$

this will give a reduced model that will be physically equivalent to that of the monopole model described above – as can be seen by making the identifications $\mathcal{X}_1 = \cos\{\theta/2\}$, $\mathcal{X}_2 = \cos\{\chi/2\}$, $\mathcal{X}_3 = \cos\{\sigma/2\}$, in the unimodular limit $|\mathcal{Z}_1| = |\mathcal{Z}_2| = |\mathcal{Z}_3| = 1$, which leads to expressions of exactly the same form (61), (62) as before. This octovac triplet model differs in principle from the monovac triplet model in that two kinks not just one are needed to obtain a complete winding. However this difference would not be perceptible in practice within the restricted framework of the reduced model, as there would be no way of telling whether the periodicity of the phase variables should be 2π or 4π , and therefore no way of knowing whether the relevant vacua were really physically distinct or not.

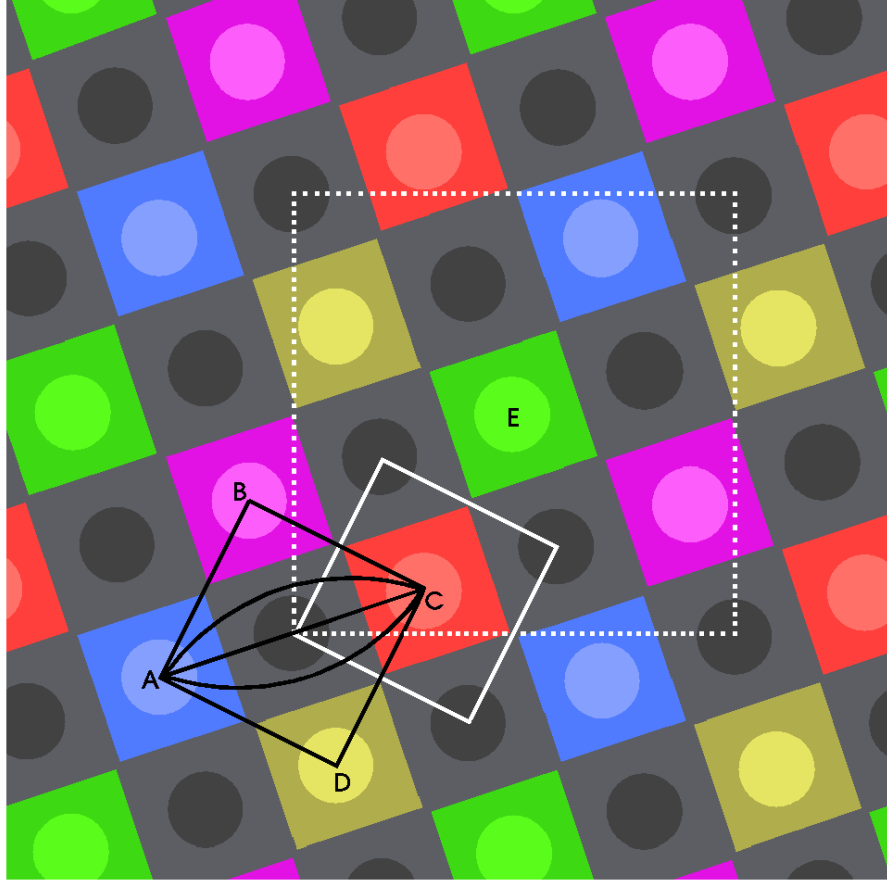


Figure 2: Contours of the potential V , using darkest shading at maxima and bright colours to distinguish distinct minima – representing vacua – labelled A, B, C, D, E, showing the approximate location of some alternative paths from A to C, in plot of ϕ against ψ representing a (fourfold) periodic covering of toroidal field configuration space of pentavac doublet model with flat kinetic metric $G_{ij} = \eta^2 \delta_{ij}$ and energy metric $\tilde{G}_{ij} = 2VG_{ij}$. The large white dotted square contains the single covering range $0 \leq \phi \leq 2\pi$, $0 \leq \psi \leq 2\pi$, and the small white square contains the range $0 \leq \theta \leq 2\pi$, $0 \leq \chi \leq 2\pi$ that would provide a single covering for the corresponding monovac doublet model, for which (as would be the shown by a black and white printout of this figure) the five kinds of vacuum colour A, B, C, D, E would cease to be distinguishable.